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**MIHAIL SANDU**

**ASTRONOMY**  
**and**  
**ASTROPHYSICS**

**MY NEW PROBLEMS**

**Volume II**



EDITURA DIDACTICĂ ȘI PEDAGOGICĂ S.A.

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## Problem 51. Spaceship to the Alpha Centauri Star

Space Shuttle leaves the Earth in a straight line and uniform motion towards the Centauri Star  $\alpha$ , with the velocity  $\vec{u}$  relative to the Earth as shown in the drawing in Figure 1. The Centauri Star  $\alpha$  is considered to be at rest relative to the Earth. Immediately after the launch of the Space Shuttle, from it, a Cosmic Rocket takes off towards the Centauri Star  $\alpha$ .

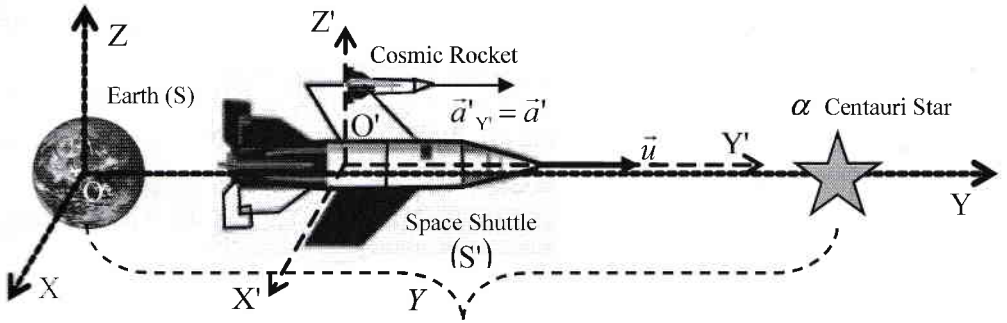


Fig. 1

The propulsion force of the Space Rocket provides it, towards the Centauri Star  $\alpha$ , a constant relative acceleration,  $\vec{a}'_{y'} = \vec{a}'$ , considered in relation to the inertial system attached to the Space Shuttle.

At the time of launch of the Space Shuttle, when the origins  $O$  and, respectively  $O'$ , of the two inertial reference systems,  $S$  and respectively  $S'$ , associated with the Earth and respectively the Space Shuttle, coincide, the clocks in the two reference systems have been synchronized, so that  $t = t' = 0$ .

The Space Rocket takes off from the Space Shuttle from the point corresponding to the origin of the inertial system associated with the Space Shuttle. The speed of light in a vacuum is known,  $c$ .

a) Establish the relationship between the acceleration of the Space Rocket in relation to the Earth,  $a_Y = a$ , and the speed of the Space Rocket in relation to the Earth,  $v_Y = v$ .

b) Establish the speed law of the Cosmic Rocket, in relation to the Earth,  $v_Y(t) = v(t)$ .

c) Establish the law of acceleration of the Cosmic Rocket, in relation to the Earth,  $a_Y(t) = a(t)$ .

d) Establish the law of the position coordinate of the Cosmic Rocket, in relation to the Earth,  $y(t)$ . It is known that:

$$\int \frac{A+B \cdot x}{C+D \cdot x} \cdot dx = \frac{1}{D} \left( B \cdot x + \frac{\Delta}{D} \cdot \ln|C+D \cdot x| \right); \Delta = AD - BC.$$

e) Determine the distance between the Earth and the Centauri Star  $\alpha$ ,  $Y$ , if the duration of the Cosmic Rocket's travel between the Earth and the Centauri Star, relative to the observer on Earth, is  $T$ .

f) Determine the speed of the Cosmic Rocket, in relation to the Earth, at the time of its arrival at the Centauri Star  $\alpha$ .

g) Determine the duration of the movement of the Cosmic Rocket between the Earth and the Centauri Star  $\alpha$ , timed by the observer  $O'$  on the Space Shuttle,  $T'$ , as well as the distance traveled by the Cosmic Rocket, in relation to the observer  $O'$  on the Space Shuttle, from the moment the Rocket took off from the Space Shuttle and until upon arrival at Centauri Star  $\alpha$ ,  $Y'$ .

h) Particularize the results from points b, c and d, for the case  $u \ll c$ , knowing that:

$$\ln(1+x) \approx x - \frac{1}{2}x^2, \text{ dac\u0103 } x \ll 1.$$

**Solving**

a) According to the notations in figure 2, corresponding to a certain moment in the evolution of the Cosmic Rocket, related to the two inertial reference systems, it follows:

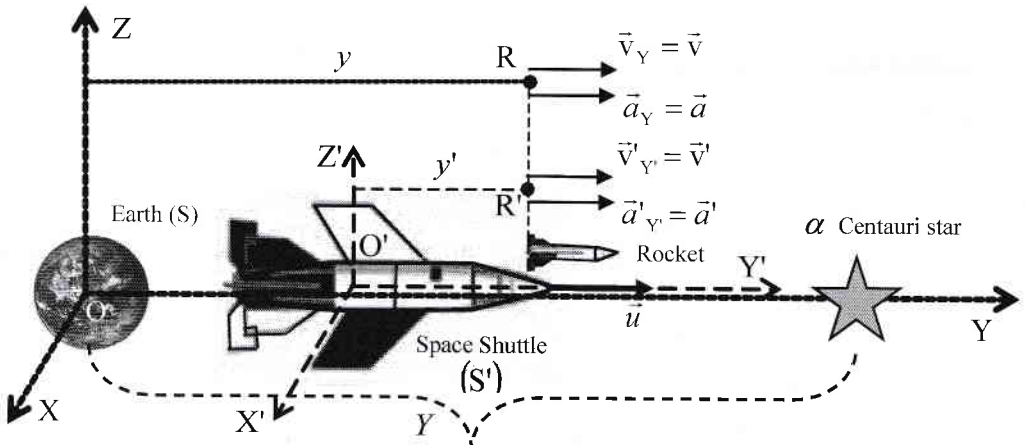


Fig. 2

$$R'(0, y', 0; t'); R(0, y, 0; t);$$

$$y = \frac{y' + ut'}{\sqrt{1 - \frac{u^2}{c^2}}} > y'; \quad t = \frac{t' + \frac{uy'}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} > t';$$

$$v_Y = \frac{dy}{dt} = \frac{\frac{dy' + u \cdot dt'}{\sqrt{1 - \frac{u^2}{c^2}}}}{dt' + \frac{u}{c^2} dy'} = \frac{dy' + u \cdot dt'}{dt' + \frac{u}{c^2} dy'} = \frac{\left(\frac{dy'}{dt'} + u\right) \cdot dt'}{\left(1 + \frac{u}{c^2} \cdot \frac{dy'}{dt'}\right) \cdot dt'};$$

$$v_Y = \frac{\frac{dy'}{dt'} + u}{\left(1 + \frac{u}{c^2} \cdot \frac{dy'}{dt'}\right)}; \quad v_Y = \frac{v'_{Y'} + u}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)};$$

$$dv_Y = \frac{d(v'_{Y'} + u) \cdot \left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right) - (v'_{Y'} + u) \cdot d\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^2};$$

$$dv_Y = \frac{dv'_{Y'} \cdot \left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right) - (v'_{Y'} + u) \cdot \frac{u}{c^2} dv'_{Y'}}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^2};$$

$$dv_Y = \frac{1 - \frac{u^2}{c^2}}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^2} \cdot dv'_{Y'};$$

$$t = \frac{t' + \frac{uy'}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}};$$

$$dt = \frac{dt' + \frac{u}{c^2} dy'}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{1 + \frac{u}{c^2} \cdot \frac{dy'}{dt'}}{\sqrt{1 - \frac{u^2}{c^2}}} \cdot dt';$$

$$dt = \frac{1 + \frac{u}{c^2} \cdot v'_{Y'}}{\sqrt{1 - \frac{u^2}{c^2}}} \cdot dt';$$

$$a_Y = \frac{dv_Y}{dt}; \quad a'_{Y'} = \frac{dv'_{Y'}}{dt'};$$

$$a_Y = \frac{\frac{1 - \frac{u^2}{c^2}}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^2} \cdot dv'_{Y'}}{\frac{1 + \frac{u}{c^2} \cdot v'_{Y'}}{\sqrt{1 - \frac{u^2}{c^2}}} \cdot dt'} = \frac{\frac{1 - \frac{u^2}{c^2}}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^2} \cdot dv'_{Y'}}{\frac{1 + \frac{u}{c^2} \cdot v'_{Y'}}{\sqrt{1 - \frac{u^2}{c^2}}}} \cdot \frac{dv'_{Y'}}{dt'};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)^3}; \quad v_Y = \frac{v'_{Y'} + u}{\left(1 + \frac{u}{c^2} \cdot v'_{Y'}\right)};$$

$$v'_{Y'} = \frac{v_Y - u}{\left(1 - \frac{u}{c^2} \cdot v_Y\right)};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y}\right)^3};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot \frac{v_Y - u}{c^2 - u v_Y} \cdot c^2\right)^3}; \quad a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u v_Y - u^2}{c^2 - u v_Y}\right)^3};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(\frac{c^2 - u v_Y + u v_Y - u^2}{c^2 - u v_Y}\right)^3};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(\frac{c^2 - u^2}{c^2 - u v_Y}\right)^3}; \quad a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(\frac{1 - \frac{u^2}{c^2}}{1 - \frac{u v_Y}{c^2}}\right)^3};$$

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u v_Y}{c^2}\right)^3}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}};$$

$$a'_{Y'} = a'; \quad a_Y = a; \quad v_Y = v; \quad a = a' \cdot \frac{\left(1 - \frac{u v}{c^2}\right)^3}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}.$$

b)

$$v_Y = \frac{u + v'_{Y'}}{1 + \frac{u}{c^2} \cdot v'_{Y'}};$$

$$v'_{Y'} = a'_{Y'} \cdot t';$$

$$v_Y = \frac{u + a'_{Y'} t'}{1 + \frac{u}{c^2} \cdot a'_{Y'} t'};$$

$$t = \frac{t' + \frac{uy'}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} > t'; \quad t' = \frac{t - \frac{uy}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} < t; \quad y = ut;$$

$$t' = \frac{t - \frac{u}{c^2} ut}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{t \left(1 - \frac{u^2}{c^2}\right)}{\sqrt{1 - \frac{u^2}{c^2}}};$$

$$t' = t \cdot \sqrt{1 - \frac{u^2}{c^2}};$$

$$v_Y = \frac{u + a'_{Y'} t'}{1 + \frac{u}{c^2} \cdot a'_{Y'} t'};$$

$$v_Y = \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} = v_Y(t); \quad a'_{Y'} = a'; \quad v_Y = v;$$

$$v_Y = \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} = v_Y(t).$$

c) From the previous equations:

$$a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u v_Y}{c^2}\right)^3}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}; \quad a_Y = a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y}\right)^3},$$

$$v_Y = \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} = v_Y(t);$$

results:

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$$\begin{aligned}
 & \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} - u \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{\frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} - u}{1 - \frac{u}{c^2} \cdot \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}}; \\
 & \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - u - \frac{u^2}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{\frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - u - \frac{u^2}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - \frac{u}{c^2} \left( \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \right)}; \\
 & \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - u - \frac{u^2}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{\frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - u - \frac{u^2}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}}{\frac{1}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - \frac{u}{c^2} \left( \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \right)}}; \\
 & \frac{a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \cdot \left( 1 - \frac{u^2}{c^2} \right)}{1} \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{\frac{a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \cdot \left( 1 - \frac{u^2}{c^2} \right)}{1}}{\frac{1}{1 - \frac{u^2}{c^2} + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - \frac{u}{c^2} \left( a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \right)}};
 \end{aligned}$$

$$\begin{aligned}
 & a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \cdot \left(1 - \frac{u^2}{c^2}\right) \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{1}{\frac{1 - \frac{u^2}{c^2}}{1}}; \\
 & \frac{1}{a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= \frac{1}{1}; \\
 \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y} &= a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t; \\
 a_Y &= a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot \frac{v_Y - u}{1 - \frac{u}{c^2} \cdot v_Y}\right)^3}; \\
 a_Y &= a'_{Y'} \cdot \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t\right)^3} = a_Y(t).
 \end{aligned}$$

d) Knowing that:

$$\begin{aligned}
 v_Y &= \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} = v_Y(t); \\
 v_Y &= \frac{dy}{dt},
 \end{aligned}$$

results:

$$y = \int v_Y dt = \int \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} \cdot dt;$$

$$\int \frac{A + B \cdot x}{C + D \cdot x} \cdot dx = \frac{1}{D} \left( B \cdot x + \frac{\Delta}{D} \cdot \ln|C + D \cdot x| \right);$$

$$\Delta = AD - BC;$$

$$A = u; B = a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}}; C = 1; D = \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}};$$

$$x = t;$$

$$\Delta = \frac{u^2}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} - a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} = -a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot \left( 1 - \frac{u^2}{c^2} \right);$$

$$\Delta = -a'_{Y'} \cdot \left( 1 - \frac{u^2}{c^2} \right)^{3/2};$$

$$y(t) = \frac{1}{\frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}}} \left( a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - \frac{a'_{Y'} \cdot \left( 1 - \frac{u^2}{c^2} \right)^{3/2}}{\frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}}} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \right| \right);$$

$$y(t) = \frac{c^2}{u} \cdot \frac{1}{a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}}} \left( a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t - \frac{c^2}{u} \cdot \frac{\left( 1 - \frac{u^2}{c^2} \right)^{3/2}}{\sqrt{1 - \frac{u^2}{c^2}}} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \right| \right);$$

$$y(t) = \frac{c^2}{u \cdot a'_{Y'}} \left( a'_{Y'} \cdot t - \frac{c^2 \cdot \sqrt{1 - \frac{u^2}{c^2}}}{u} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \right| \right);$$

$$y(t) = \frac{c^2}{u} \left( t - \frac{c^2 \cdot \sqrt{1 - \frac{u^2}{c^2}}}{u \cdot a'_{Y'}} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t \right| \right)$$

e)

$$t = T; y(t) = y(T) = Y;$$

$$y(t) = \frac{c^2}{u \cdot a'_{Y'}} \left( a'_{Y'} \cdot t - \frac{c^2 \cdot \sqrt{1 - \frac{u^2}{c^2}}}{u} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot t \cdot \sqrt{1 - \frac{u^2}{c^2}} \right| \right)$$

$$Y = \frac{c^2}{u \cdot a'_{Y'}} \left( a'_{Y'} \cdot T - \frac{c^2 \cdot \sqrt{1 - \frac{u^2}{c^2}}}{u} \cdot \ln \left| 1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot T \cdot \sqrt{1 - \frac{u^2}{c^2}} \right| \right)$$

f)

$$t = T; v_Y(t) = v_Y(T) = v_{Y,\max};$$

$$v_Y = \frac{u + a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot \sqrt{1 - \frac{u^2}{c^2}} \cdot t} = v_Y(t);$$

$$v_{Y,\max} = \frac{u + a'_{Y'} \cdot T \cdot \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c^2} \cdot a'_{Y'} \cdot T \cdot \sqrt{1 - \frac{u^2}{c^2}}}$$

g)

$$Y' = \frac{a'_{Y'} T^2}{2}; Y = \frac{Y' + uT'}{1 + \frac{u v'_{Y',\max}}{c^2}}; v'_{Y',\max} = a'_{Y'} T';$$

$$Y = \frac{Y' + uT'}{1 + \frac{u a'_{Y'} T'}{c^2}}; Y = \frac{\frac{a'_{Y'} T'^2}{2} + uT'}{1 + \frac{u a'_{Y'} T'}{c^2}};$$